International Journal of Theoretical Physics, Vol. 29, No. 12, 1990

Anomalous Parity-Violating Coupling of Photon to Nucleon

P. Żenczykowski[1,2]

Received June 16, 1990

It is pointed out that in the naive quark model the weak-interaction-induced parity-violating coupling of transverse photons to the nucleon does not vanish at $q^2 = 0$. The presence of this nonvanishing "anomalous" parity-violating $NN\gamma$ coupling g_a is a direct consequence of the *composite* nature of hadrons: gauge invariance requires the vanishing of g_a only if the nucleon can be described by a Dirac spinor depending on a *single* spacetime point. Nonvanishing of g_a in the quark model makes precise measurements of weak radiative hyperon decays particularly important.

In a recent paper (Żenczykowski, 1989) it was shown that the vector dominance model (VDM), which at $q^2 = 0$ is essentially equivalent (Sakurai, 1969) to the naive quark model (NQM), indicates the presence of a non-vanishing parity-violating (p.v.) $\Sigma p\gamma$ coupling of the effective form $F(q^2 = 0)u_p\gamma_\mu\gamma_5 u_{\Sigma^+}A^\mu$. In this way VDM sheds some light on the origin of the nonvanishing of the p.v. $\Sigma^+ \to p\gamma$ decay amplitude in the NQM (Kamal and Riazuddin, 1983; also see Hara, 1964). From flavor symmetry one expects then that also the diagonal γNN p.v. coupling, contrary to hadron-level standard arguments, should be nonzero in the quark model. In this paper we show that this is indeed the case and we exhibit the origin of the difference between NQM calculations and hadron-level expectations.

Let us first recall the standard hadron-level argument requiring the vanishing of the coupling in question. Because we are dealing with ($N \to N\gamma$) transitions diagonal in flavor, we write the most general CP-conserving, p.v.

[1]Department of Physics, University of Guelph, Guelph, Ontario, Canada N1G 2W1.
[2]On leave of absence from the Department of Theoretical Physics, Institute of Nuclear Physics, Cracow, Poland.

0020-7748/90/1200-1327$06.00/0

photon–nucleon–nucleon coupling at the hadron level as

$$\bar{u}_N(p_f)(F_1(q^2)\gamma_\mu\gamma_5 + F_2(q^2)q_\mu\gamma_5)u_N(p_i)\varepsilon^\mu \qquad (q = p_f - p_i) \tag{1}$$

The current in (1) has $C = +1$, $P = +1$. At this stage the requirement of gauge invariance is imposed, leading to the condition $2m_N F_1(q^2) + q^2 F_2(q^2) = 0$. The absence of a U-spin singlet, exactly massless particle (i.e., no pole in F_2 at $q^2 = 0$) demands then that $F_1(0) = 0$.[3]

Consider now the NQM contribution to the parity-violating $NN\gamma$ coupling g_a shown (for N = proton) in Figure 1a. In the naive quark model this contribution to the coupling in question is defined as the expectation value of the operator of Figure 1a (plus the contributions from diagrams in which the order of photon and weak boson emissions from the first quark line is interchanged, plus terms with different quark orderings: $udu \leftrightarrow duu$) sandwiched between $SU(6)$ wave functions of the NQM.[4] We are interested in the properties of the "composite current" relevant to that coupling (i.e., when A_μ is factored out) under the operation of charge conjugation C. To obtain a charge-conjugated process from Figure 1a, we must replace quarks

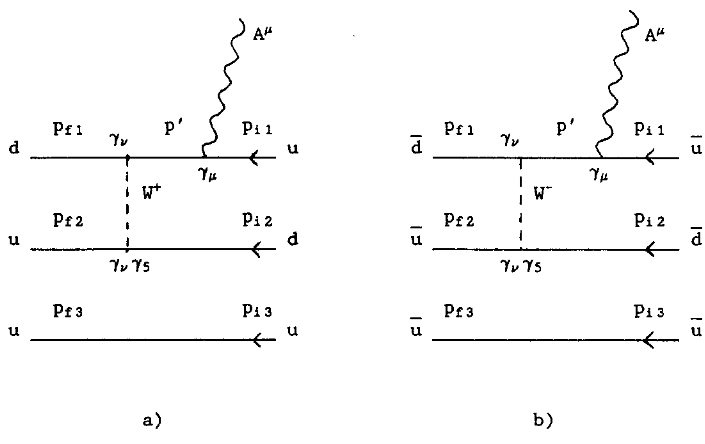

Fig. 1. The W-exchange diagrams contributing to the p.v. coupling of photons to the nucleon in the quark model.

[3]In the nonrelativistic limit the resulting interaction has the form $a * \sigma * \text{curl}\,\mathbf{H}$, where a is the so-called anapole moment first considered in Zeldovich (1957; Zeldovich and Perelomov, 1960). Even for the leptons the question of the observability of the anapole moment may be considered open (Czyz *et al.*, 1988). In this paper, however, we are interested in the coupling itself and not in the moment.

[4]Due to the symmetry of the spin-flavor wave function, it suffices to calculate the contribution from diagrams in which a photon is attached to the first quark line and the W-boson is exchanged between first and second quark.

by antiquarks without changing anything else. The effect of the inclusion of antiquarks is best seen if we write the antiquark spinors in the "constituent" currents in the particlelike representation, i.e., $u_{\bar{q}}(p) = -i\gamma_2\gamma_0[\bar{v}(-p)]^T$. In momentum representation the relevant currents are

$$\bar{u}_u(p')\gamma_\mu u_u(p_{i1}) - \bar{u}_{\bar{u}}(p')\gamma_\mu u_{\bar{u}}(p_{i1}) \tag{2a}$$

$$\bar{u}_d(p_{f1})\gamma_\nu u_u(p') - \bar{u}_{\bar{d}}(p_{f1})\gamma_\nu u_{\bar{u}}(p') \tag{2b}$$

$$\bar{u}_u(p_{f2})\gamma_\nu\gamma_5 u_d(p_{i2}) + \bar{u}_{\bar{u}}(p_{f2})\gamma_\nu\gamma_5 u_{\bar{d}}(p_{i2}) \tag{2c}$$

The minus sign in equation (2a) represents then the opposite charge of an antiquark as compared to that of a quark. When the operation of charge conjugation ($q \to \bar{q}$) is applied to expression in equation (2a), one obtains the same expression with a negative sign which, when the requirement of C-parity conservation is imposed, admits its coupling to the photon.

The proper parity-violating coupling of the photon A^μ to the nucleon(in)-nucleon(out) plus antinucleon(in)-antinucleon(out) system is given by the sum of expressions corresponding to Figures 1a and 1b. Indeed, from equations (2) we see that due to the presence of two negative signs in equations (2a), (2b) the relevant "composite NQM current" is C-parity even (i.e., it reproduces itself under the replacement $q \to \bar{q}$, all q's). Needless to say, the current is even under ordinary reflection. The "composite NQM current" represented by the sum of Figures 1a and 1b thus has $C = +1$, $P = +1$, as the standard hadron-level current in equation (1). It is clear in this formulation that the consideration of CP properties at the quark (NQM) level plays no role whatsoever in determining the behavior of the parity-violating nucleon-photon coupling at zero photon momentum. Vanishing or nonvanishing of that coupling for identical incoming and outgoing baryons is decided by the form of the diagram of Figure 1a. To know whether or not it vanishes one only needs to perform a simple calculation.

The actual calculations may be done in two ways. One can either perform the calculations in the naive quark model itself following Kamal and Riazuddin (1983) or one can use the vector-meson dominance model as in Żenczykowski (1989). The two approaches are essentially equivalent at $q^2 = 0$ (Żenczykowski, 1989; Sakurai, 1969). In the quark-model framework the calculation proceeds exactly like that of the $\Sigma^+ \to p\gamma$ (i.e., $suu \to duu + \gamma$) weak radiative hyperon decay [in the $SU(3)$ symmetry limit] presented in Kamal and Riazuddin (1983), so there is no need to repeat it here. The only differences when compared to that reference are (i) the change of name of one of the initial quarks ($s \to d$) and (ii) the difference in the overall (multiplicative) Cabibbo factors. When the $SU(6)$ spin-flavor (NQM) wave functions are assumed for the nucleons, the calculation shows that the parity-violating nucleon-photon coupling does *not* vanish at zero photon momentum. This is corroborated by a VDM argument in which the effective coupling of a photon to hadrons is obtainable from that of a vector meson by the substitution $V_\mu \to (e/g_V)A_\mu$ (Żenczykowski, 1989; Sakurai, 1969). Since no general principle requires the vanishing of the p.v. coupling of the nucleon to a U-spin singlet transverse vector meson, one again obtains a nonzero p.v. photon–nucleon coupling.

The quark model and VDM predict therefore the existence of a non-vanishing "anomalous" parity-violating $NN\gamma$ coupling g_a induced by the W- (and Z-) exchange processes inside the nucleon. Its size, in complete analogy to the calculations of parity-violating amplitudes in weak radiative hyperon decays (Żenczykowski, 1989), is best estimated from VDM ideas. Using parity-violating proton–vector meson couplings from Desplanques *et al.* (1980) and the vector-meson dominance prescription, we obtain for the proton [in the $SU(3)$ limit]

$$\begin{aligned} g_a(p) = e/g * \sqrt{2}/3\{&[(-b+c-4a/3)\cot\theta_C + 8/9*(c+a)\tan\theta_C] \\ &+\csc(2\theta_C)*[-b(1-(16/9)\sin^2\theta_W) \\ &+c(1-(68/27)\sin^2\theta_W)+12a(1-(184/81)\sin^2\theta_W)]\} \end{aligned} \quad (3)$$

where θ_C and θ_W are the Cabibbo and Weinberg angles, $g = 5.0$ is the $g_{\rho NN}$ coupling, and the parameters a, b, c were estimated in Żenczykowski (1989) through $SU(6)_W$ symmetry from nonleptonic hyperon decays to be $b = -5.0$, $c = 12.0$, $a = 1.0$ (in units of 10^{-7}). In equation (3) the size of contribution from diagrams of Figure 1 with two-quark interactions is described by a single parameter (b). The remaining two parameters (a, c) present in equation (3) correspond to two types of single-quark diagrams whose nonvanishing contribution is required by $SU(6)_W$ arguments and experimental data on nonleptonic hyperon decays (Żenczykowski, 1989). The term proportional to $\csc(2\theta_C)$ is due to Z^0 exchange. The resulting value is $g_a(p) \approx 9.6 * 10^{-7}e$ ($W^\pm$-exchange contributes $6.6 * 10^{-7}e$). While the values of parameters b, c are determined from nonleptonic hyperon decays with good accuracy, the value of a is less reliable. If $a = 1.3$ is used (this value describes the $\Xi^- \to \Sigma^-\gamma$ branching ratio a little better than $a = 1.0$), we obtain instead $g_a(p) = 9.8 * 10^{-7}e$, which does not differ significantly from our previous estimate. If, on the other hand, one sets $a = c = 0$ so that only the contributions from the W- (and Z-) exchange diagrams of Figure 1 are considered, one obtains $g_a(p) = 2.6 * 10^{-7}e$.

The NQM/VDM calculations are in an apparent disagreement with the hadron-level arguments requiring the vanishing of the coupling in question. In fact, VDM explicitly indicates the existence of an effective $F_1(q^2=0)\bar{u}\gamma_\mu\gamma_5 u A^\mu$ coupling that does not vanish at zero photon momentum. Clearly, somewhere there must be an essential difference between the quark model approach and the hadron-level standard arguments. This difference is pointed out below.

Consider the NQM in which the nth quark ($n=1,2,3$) is described by the Dirac equation (equal-mass case):

$$\hat{H}_n q_n(x_n) = [\boldsymbol{\alpha}_n \cdot \hat{\mathbf{p}}_n + m\beta_n] q_n(x_n) = i\frac{\partial}{\partial x_n^0} q_n(x_n) \tag{4}$$

We assume the $SU(6)$ wave functions for the nucleon

$$\Psi_D(x_1, x_2, x_3) = \sum C_D^{ABC} q_{1\alpha}(x_1) q_{2\beta}(x_2) q_{3\gamma}(x_3)$$

where A, B, C, D are spin and flavor indices of nonrelativistic $SU(6)$ corresponding to $SU(12)$ [Dirac and $SU(3)$] indices $\alpha, \beta, \gamma = (A, a), (B, b), (C, c)$ with $a, b, c = 1, 2$. Thus, apart from the spin-flavor index, Ψ has an additional set of indices $[a, b, c]$ resulting from the existence of positive- and negative-parity components in quark Dirac spinors.

From the additivity of the NQM we obtain

$$\hat{H}\Psi \equiv \sum_n \hat{H}_n \Psi(x_1, x_2, x_3) = i\frac{\partial}{\partial X^0} \Psi(x_1, x_2, x_3) \tag{5}$$

where $X^\mu = (x_1^\mu + x_2^\mu + x_3^\mu)/3$.

Using plane waves for the quarks and factoring out the relative wave, we rewrite the above equation as

$$i\frac{\partial}{\partial X^0}\tilde{\Psi}(X) = \left[\tfrac{1}{3}\sum_n \tilde{\boldsymbol{\alpha}}_n \cdot \hat{\mathbf{P}} + \tfrac{1}{2}(\tilde{\boldsymbol{\alpha}}_1 - \tilde{\boldsymbol{\alpha}}_2)\cdot(\mathbf{p}_1 - \mathbf{p}_2) + \tfrac{1}{6}(\tilde{\boldsymbol{\alpha}}_1 + \tilde{\boldsymbol{\alpha}}_2 - 2\tilde{\boldsymbol{\alpha}}_3)\right.$$
$$\left. \times (\mathbf{p}_1 + \mathbf{p}_2 - 2\mathbf{p}_3)\right]\tilde{\Psi}(X) + m\sum_n \tilde{\beta}_n \tilde{\Psi}(X) \tag{6}$$

where $\hat{\mathbf{P}}$ is the operator of the total momentum of the nucleon and

$$\tilde{\boldsymbol{\alpha}}_1 = \boldsymbol{\alpha}_1 \otimes 1_2 \otimes 1_3$$

$$\tilde{\boldsymbol{\alpha}}_2 = 1_1 \otimes \boldsymbol{\alpha}_2 \otimes 1_3$$

$$\tilde{\boldsymbol{\alpha}}_3 = 1_1 \otimes 1_2 \otimes \boldsymbol{\alpha}_3$$

where the factors in the direct products act in spinor spaces of the first, second, and third quarks, respectively. Similar expressions are easily written down for $\tilde{\beta}_n$.

One immediately checks that the matrices $\tilde{\boldsymbol{\alpha}} \equiv \frac{1}{3}\sum_n \tilde{\boldsymbol{\alpha}}_n$, $\tilde{\beta} \equiv \frac{1}{3}\sum_n \beta_n$ do not satisfy algebraic relations characteristic of the Dirac equation. This fairly trivial result constitutes the formal reason why the standard argumentation leading to the conclusion $F_1(0)=0$ cannot be applied in the NQM framework. In its essence the NQM contribution constitutes a scattering amplitude of free quarks and as such does not correspond in any way to the formula of (1) in which the external state is a single *pointlike* nucleon. Indeed, by writing hadron level couplings in the form of (1), we treat nucleons in *exactly the same way* as (pointlike) electrons.

From equation (5) and the subsequent discussion it follows that there are two related features that distinguish the quark- and the standard hadron-level approaches. The first is the dimensionality of the wave function assigned to the nucleon, the second is the number and meaning of wave function arguments. One can easily see how the latter point affects the way in which the concept of gauge invariance is introduced. In the standard hadron-level description in momentum representation one considers a nucleon to be fully described by a Dirac spinor corresponding to hadron total momentum p. This spinor constitutes the coefficient in the plane wave representation of the spinor field $\psi(x)$ depending on a *well-defined single* point x at which the gauge transformations of the hadron level are carried out. The applicability of gauge transformations at this point does not carry through to the quark level: the quarks are not located at x, which describes the position of the nucleon as a whole and is conjugate to the sum of quark momenta $p_1+p_2+p_3$, but at different positions x_1, x_2, x_3 each of which is separately conjugate to the corresponding momentum. In the quark-level description gauge transformations at the nucleon center of mass $x=(x_1+x_2+x_3)/3$ have no meaning at all. Similar arguments apply in momentum space: from the knowledge of total hadron momentum *only*, one cannot retrieve the momenta of individual quarks, a necessary procedure if quark fields in position representation (needed for a discussion of gauge invariance at the quark level) are to be considered. The only possibility left is to discuss gauge invariance at the hadron level in which one considers the nucleon as a pointlike particle. Thus, if the nucleon is described by a local Dirac field, some of the quark level information is lost. In the naive quark model, hadrons possess *multilocal*[5] structure. Consequently, the appropriate hadron-level field-theoretic language is that of an effective multilocal field theory in which a nucleon would be described by a multispinor field $\psi_{\alpha\beta\gamma}(x_1, x_2, x_3)$, etc. The multilocal nature of the quark model (and its structural difference from local field theory) is not generally appreciated, however.

[5]Nonlocal field theory of this type was originally considered in Yukawa (1950).

The appearance of the "anomalous" parity-violating $NN\gamma$ coupling in the NQM might be regarded as an artefact of the naive quark model. On the other hand, the presence of this term stems from the same assumptions that were so successful elsewhere, e.g., in the quark model prediction for the anomalous magnetic moment of the neutron. The essential ingredient in all such calculations is the *compositeness* assumption through which pointlike quarks are correlated in a quantum mechanical way by the $SU(6)$ wave function. Furthermore, nonvanishing of the coupling in question is corroborated by vector dominance model, which is well known for its reliability. Therefore, I find it difficult to believe that the quark model prediction of the nonvanishing g_a is just an artefact of the quark model. Indeed, the data on weak radiative decays of hadrons seem to corroborate the NQM/VDM predictions. In particular, the most recent data (James *et al.*, 1990) on the $\Xi^0 \to \Lambda\gamma$ asymmetry seem to indicate its positiveness, in agreement with NQM/VDM expectations (Żenczykowski, 1989; Verma and Sharma, 1988) and in disagreement with previous calculations in which the effective $\bar{u}\gamma_\mu\gamma_5 u A^\mu$ term was absent. The recently measured $\Xi^0 \to \Sigma^0\gamma$ branching ratio (Teige *et al.*, 1989) is also in agreement with NQM/VDM predictions. On the other hand, the corresponding asymmetry parameter seems to be in conflict both with NQM/VDM and with all other theoretical estimates.

Although from the technical point of view the origin of the difference between the standard hadron-level arguments and the NQM/VDM calculations is obvious, deeper theoretical resolution of various emerging questions might have far-reaching implications. In view of the great importance of questions related to the meaning and/or applicability of quark model prescriptions, it is therefore crucial that the p.v. amplitudes in weak radiative hyperon decays be precisely measured.

ACKNOWLEDGMENT

I would like to thank Prof. G. Karl for very interesting discussions and for reading the manuscript.

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